Rate of Return Analysis

- Recall the $5,000 debt example in chapter 3.
- Each of the four plans were used to repay the amount of $5000.
- At the end of 5 years, the principal and interest payments exactly repaid the $5000 debt with 8% interest.
- The total interest paid to the lender varied from $1200 to $2347.
- Despite this variation in total interest paid, we say that the lender received an 8% rate of return.
- Since we are describing situations of funds that remain within the investment throughout its life, the resulting rate is called the internal rate of return, *i*.

Calculating Rate of Return

- Convert the various consequences of the investment into a cash flow.
- Solve the cash flow for the unknown value of the internal rate of return (IRR).
- Use any of the following forms:
  1. PW of benefits – PW of costs = 0
  2. (PW of benefits)/(PW of costs) = 1
  3. Net present worth = 0
  4. EUAB – EUAC = 0
  5. PW of costs = PW of benefits
- Any of the previous forms relate costs and benefits with the IRR as the only unknown.
- **Rate of Return**: It is the interest rate at which the benefits are equivalent to the costs

Example 7-1

An $8200 investment returned $2000 per year over a 5-year useful life. What was the rate of return on this investment?

(PW of benefits)/(PW of costs) = 1

2000*(P/A, i, 5)*/8200 = 1

(P/A, i, 5) = 8200/2000 = 4.1

Look at the compound interest tables for the value of *i* where *(P/A, i, 5)* = 4.1. If no tabulated value exists, use interpolation.

From the interest tables, one can find that for *(P/A, i, 5)* = 4.1, *i* is 7%. (no interpolation was needed)

The rate of return is exactly <u>7%</u>.

Example 7-2

An investment resulted in the following cash flow. Compute the rate of return.

Year	Cash Flow
0	–\$700
1	+100
2	+175
3	+250
4	+325

EUAB – EUAC = 0

$100 + 75(A/G, i, 4) - 700(A/P, i, 4) = 0$

Solve the equation by trial and error since we two unknown interest factors.

Try $i = 5\%$ ►► $100 + 75(1.439) - 700(0.2820) = +11$

Try $i = 8\%$ ►► $100 + 75(1.404) - 700(0.3019) = -6$

Try $i = 7\%$ ►► $100 + 75(1.416) - 700(0.2952) = 0$

Therefore the IRR is exactly <u>7%</u>. (Again, no interpolation was needed)

Example 7-3

Year	Cash Flow
0	–$100
1	+20
2	+30
3	+20
4	+40
5	+40

Calculate the rate of return on the investment on the following cash flow.

Use the form: NPW = 0

NPW = –100 + 20*(P/F, i, 1)* + 30*(P/F, i, 2)*
+ 20*(P/F, i, 3)* + 40*(P/F, i, 4)* + 40*(P/F, i, 5)*

Try *i* = 10% ►► NPW = –100 + 20*(P/F, 10%, 1)* + 30*(P/F, 10%, 2)*
+ 20*(P/F, 10%, 3)* + 40*(P/F, 10%, 4)* + 40*(P/F, 10%, 5)*

NPW = –100 + 20*(0.9091)* + 30*(0.8264)* + 20*(0.7513)* + 40*(0.6830)*
+ 40*(0.6209)* = +10.16

Try *i* = 12% ►► NPW = –100 + 20*(0.8929)* + 30*(0.7972)* + 20*(0.7118)*
+40*(0.6355)* + 40*(0.5674)* = +4.126

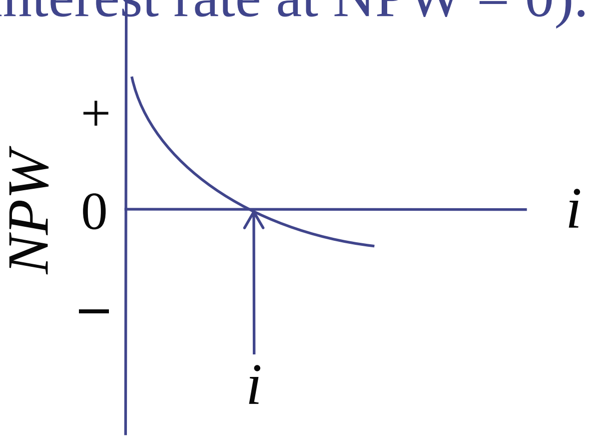
Try *i* = 15% ►► NPW = –100 + 20*(0.8969)* + 30*(0.7561)* + 20*(0.6575)*
+40*(0.5718)* + 40*(0.4972)* = – 4.02

Therefore, the IRR lies between 12% and 15%. By linear interpolation, we find that the IRR is: IRR = 13.5%

Plot of NPW versus interest rate i:

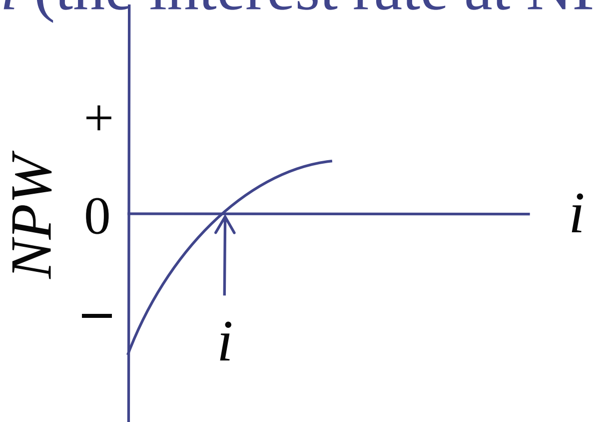
For a cash flow representing investment followed by benefits from the investment, the plot of NPW versus *i* will decrease at a decreasing rate and have a zero value at unique value *i* (the interest rate at NPW = 0).

Year	Cash Flow
0	–P
1	+benefit A
2	+A
3	+A
.	.
.	.
.	.



For borrowed money, the NPW plot will increase at a decreasing rate and have a zero value at unique value *i* (the interest rate at NPW = 0).

Year	Cash Flow
0	+P
1	– Repayment A
2	– A
3	– A
.	.
.	.



Example 7-4

A new corporate bond was initially sold by a stockbroker to an investor for $1000. The issuing corporation promised to pay the bondholder $40 interest on the $1000 face value of the bond every 6 months, and to repay $1000 at the end of 10 years. After one year the bond was sold by the original buyer for $950.

a) What rate of return did the original buyer receive on his investment?

b) What rate of return can the new buyer (paying $950) expect to receive if he keeps the bond for its remaining 9-year life?

Since $40 is received each 6 months, we will solve the problem using a 6-month interest period.

Let PW of cost=PW of benefits, and. write

1000 = 40(P / A, i, 2) + 950(P / F, i, 2) Try *i =* 11/2%:

1000 = 40(1.956) + 950(0.9707) = 78.24 + 922.17 = 1000.41

The interest rate per 6 months, IRR_{6mon}, is very close to 1½%*. This means the nominal (annual)* interest rate is 2 x 1.5%=3%.

Given the same \$40 semiannual interest payments, for 6-month interest periods we write

950 = *40(P / A, i, 18) + 1000(P/ F, i, 18)*

Try *i =5%:*

950?= 40(11.690) + 1000(0.4155)=467.60+415.50 =883.10

The PW of benefits is too low. Try a lower interest rate, say, *i =4%:*

950 =? 40(12.659) 4-1000(0.4936)=506.36+493.60=999.96

The value of the IRR is between 4 and 5%. By interpolation

IRR= 4%+ (1%) (999.96 -950.00)/(999.96 -883.10) = 4.43%

The nominal interest rate is 2 x 4.43 %= 8.86%.

Rate of Return Analysis

7.6 Motivating example

You must select one of two mutually exclusive alternatives.(Note:Engineering economists often use the term "mutually exclusive alternatives" to emphasize that selecting one precludes selecting any other alternative.)The alternatives are as follows:

Year	Alt. 1	Alt. 2
0	–$10	–$20
1	+15	+28

Which alternative would you select?

a) Using PW analysis

b) Using rate of return

Use a Minimum Attractive Rate of Return (MARR) of 6%.

a) Using PW analysis:

Alt.1: NPW = 15(P/F, 6%, 1) – 10 = 15(0.9434) – 10 = 4.15

Alt. 2: NPW = 28(P/F, 6%, 1) – 20 = 28(0.9434) – 20 = 6.415

Based on the PW analysis one should select Alt. 2.

…Rate of Return Analysis

Year	Alt. 1	Alt. 2
0	–$10	–$20
1	+15	+28

b) Using Rate of Return:

Alt. 1: PW of cost of Alt. 1 = PW of benefit of Alt. 1

$$10 \quad = \quad 15(1+ i)^{-1} \quad ►► \quad i = 50\%$$

i.e. the rate of return for Alt. 1 is 50%

Alt. 2: PW of cost of Alt. 2 = PW of benefit of Alt. 2

$$20 \quad = \quad 28(1+ i)^{-1} \quad ►► \quad i = 40\%$$

i.e. the rate of return for Alt. 2 is 40%

Based on the rate of return results one should select alternative 1, which contradicts with the PW analysis results.

What should we follow; the PW analysis or the rate of return?

…Rate of Return Analysis

Since we know that the PW analysis is correct, the previous example tells us that something went wrong in the rate of return approach.

…Rate of Return Analysis

Consider the following statements about a project:

1. The net present worth of the project is $32,000.
2. The equivalent uniform annual benefit is $2,800.
3. The project will produce a 23% rate of return

The third statement is perhaps most widely understood since it gives a measure of desirability of the project in terms that are readily understood.

Therefore the Rate of return analysis is probably the most frequently used analysis

Another advantage to rate of return analysis is that no interest rate is introduced in the calculations. Whereas both PW and annual cash flow methods require the use of an interest rate, which might be a difficult and controversial point.

...Rate of Return Analysis

For one alternative:

To determine the desirability of one alternative:

1. Compute the IRR from the cash flow
2. Compare the computed IRR with a preselected minimum attractive rate of return (MARR). The project is desirable if IRR≥MARR.

…Rate of Return Analysis

When there are two alternatives, rate of return analysis is performed by computing the incremental rate of return ΔIRR on the difference between the alternatives. The difference can be:

1. Increments of Investment:

   Compute the ΔIRR on the increment of investment between the alternatives (i.e. the cash flow for the difference between the alternatives), which is computed by taking the higher initial-cost alternative minus the lower initial-cost alternative.

   - Choose the higher-cost alternative if ΔIRR ≥ MARR
   - Choose the lower-cost alternative if ΔIRR < MARR

2. Increments of Borrowing:

   The reverse is done.

I perfer that you always do increment of investment to minimize the confusion.

…Rate of Return Analysis

Now let's go back to the motivating example:

Year	Alt. 1	Alt. 2	Alt.2 – Alt.1
0	–\$10	–\$20	–\$20 – (–\$10) = –\$10
1	+15	+28	+28 – (+15) = +13

For the cost of differences cash flow (Alt.2 – Alt.1), compute the IRR: $10 = 13(P/F, i, 1)$

$$(P/F, i, 1) = 0.7692$$

From the compound interest tables $i = 30\%$. (OR you can just say an increase of \$10 to \$13 is 30% increase)

Since $\Delta IRR \geq MARR$, choose Alternative 2.

What happened was that the 30% rate of return on the difference between the alternatives is far higher than the 6% MARR. In other words, the additional \$10 investment (at 30% IRR) is superior to investing the \$10 elsewhere at 6%.

Example 7-8

Solve the previous example using the increment Alt.1 – Alt.2:

Year	Alt. 1	Alt. 2	Alt.1 – Alt.2
0	–$10	–$20	–$10 – (–$20) = +$10
1	+15	+28	+15 – (+28) = –13

For the cost of differences cash flow (Alt.1 – Alt.2), compute the IRR: $10 = 13(P/F, i, 1)$

$(P/F, i, 1) = 0.7692$ ►► $i = 30\%$

The cashflow (Alt.1 – Alt.2) represents a loan with a 30% interest rate. We know that the MARR is 6%, which also can be assumed our maximum interest rate on borrowing. We know that a 6% loan is preferred over a 30% loan. Therefore, the increment (Alt.1 – Alt.2) is undesirable.

►► Reject Alt.1 and select Alt.2

Example 7-9

Year	Device A	Device B
0	–$1000	–$1000
1	+300	+400
2	+300	+350
3	+300	+300
4	+300	+250
5	+300	+200

This problem has been solved before by the PW analysis (example 5-1)

Year	Device A	Device B	Device A –Device B
0	–$1000	–$1000	$0
1	+300	+400	–100
2	+300	+350	–50
3	+300	+300	0
4	+300	+250	+50
5	+300	+200	+100

……

EUAC = EUAB

[100(P/ F, i, 1) + 50(P/ F, i, 2)](A/P, i, 5)= [50(F/ P, i, 1) + 100](A/ F, i, 5)

The equation need not be solved. Instead, we observe that the sum of the costs (-100 and -50) equals the sum of the benefits (+50 and + 100).

This indicates that 0% is the IRR_{A-B} on the A - B increment of investment.

This is less than the 7% MARR;

therefore, the increment is undesirable. Reject Device A and choose Device B.

As described in Example7-8,if the increment examined is (B - A),the interest rate would again be 0%, indicating a desirable borrowing situation.

We would choose Device B.

Analysis Period

Example 7.10

Two machines are being considered for purchase. If the MARR is 10%,which machine should be bought? Use an IRR analysis comparison.

	Machine X	Machine Y
Initial cos	$200	$700
Uniform annual benefit	95	120
End of Useful live salvage value	50	150
Useful life (years)	6	12

PW of cost of differences = PW of benefits of differences

500 = 25(P/A, i, 12) + 150(P/F, i, 6) + 100(P/F, i, 12)

The sum of benefits over the 12 years is $550 which is only a little greater than the $500 benefits, indicating that the rate of return is low.

Try I = 1 % ►► 25(P/A, i, 12) + 150(P/F, i, 6) + 100(P/F, i, 12) = 511

Try I = 1.5% ►►25(P/A, i, 12) + 150(P/F, i, 6) + 100(P/F, i, 12) = 494

i.e the IRR on the Y-X increment (IRR_{Y-X}) is between 1% and 1.5% which is fare below the MARR. Therefore X is preferred.

Year	machine x	machine y	machine y- machine x
0	-200	-700	-500
1	+95	+120	+25
2	+95	+120	+25
3	+95	+120	+25
4	+95	+120	+25
5	+95	+120	+25
6	+95	+120	+25
	+50		+150
	-200		
7	+95	+120	+25
8	+95	+120	+25
9	+95	+120	+25
10	+95	+120	+25
11	+95	+120	+25
12	+95	+120	+25
	+50	+150	+100

=+550(benefits)

Home work

- Chapter 6
- 6.2
- 6.5
- 6.7
- 6.12
- 6.20
- 6.23
- 6.36
- 6.40
- 6.41
- 6.49

Chapter 7

- 7.2
- 7.6
- 7.8
- 7.10
- 7.17
- 7.23
- 7.46
- 7.47
- 7.52